Indices and Logarithm

CHAPTER 5 : INDICES AND LOGARITHMS

Topic 5 Indices and Logarithm:::::::::: v 5.1 Indices ::::::::: v 5.11 Indices

1) $x^3 \times x^2 = x^{3+2} = x^5 \qquad \Rightarrow x^3 \times x^2 = xxx \times xx = xxxxx$

2) $x^7 \div x^2 = x^{7-2} = x^5 \qquad \Rightarrow x^7 \div x^2 = \frac{xxxxx\not{x}\not{x}}{\not{x}\not{x}} = xxxxx$

e.g.1:

Solve the equation $3^{2x+1} + 3^2 = 3^{x+3} + 3^x$.

Solution:

$3^{2x} \cdot 3^1 + 9 = (3^x) \cdot 3^3 + (3^x)$

$(3^x)^2 \cdot 3^1 + 9 = (3^x) \cdot 3^3 + (3^x)$

Let $3^x = u$,

$3u^2 + 9 = 27u + u$

$3u^2 + 9 = 28u$

$3u^2 - 28u + 9 = 0$

$(3u - 1)(u - 9) = 0$

$u = \frac{1}{3} @ 9$

$3^x = \frac{1}{3}$	$3^x = 9$
$3^x = 3^{-1}$	$3^x = 3^2$
$x = -1$	$x = 2$

e.g.2:

Solve the equation $2^{2x+3} + 2^{x+3} = 1 + 2^x$.

Solution:

$2^{2x+3} + 2^{x+3} = 1 + 2^x$

$2^{2x} \cdot 2^3 + 2^x \cdot 2^3 = 1 + 2^x$

$8(2^x)^2 + 8(2^x) = 1 + (2^x)$

Let $2^x = u$,

$8u^2 + 8u = 1 + u$

$8u^2 + 7u - 1 = 0$

$(8u - 1)(u + 1) = 0$

$u = -1 @ \frac{1}{8}$

$2^x = -1 (rejected)$	$2^x = \frac{1}{8}$
	$2^x = 2^{-3}$
	$x = -3$ #

***Index can not negative:
$2^3 = 8 > 0$
$2^0 = 1 > 0$
$2^{-3} = \frac{1}{8} > 0$

Topic 5 Indices and Logarithm:::::::::: v 5.1 Indices ::::::::: v 5.12 Multiplication Of Indices

1. $(x^m)^n = x^{mn}$
2. $a^m \times a^n = a^{m+n}$
3. $a^m \div a^n = a^{m-n}$
4. $(x^2)^3 = x^{2\times 3} = x^6$
5. $\sqrt{2} \times \sqrt{3} = \sqrt{6}$
6. $2^x \times 3^x = 6^x$
7. $\frac{8^x}{2^x} = 4^x$

Indices and Logarithm

Topic 5 Indices and Logarithm:::::::::: v 5.1 Indices :::::::: v 5.13 Same Indices

1. $2^2 \times 3^2 = (2\times 3)^2$
$= 6^2$
$= 36$
If the power is same, then we can multiply directly.

2. $(xy)^2 = x^2 y^2$

3. $2^x \bullet 3^{x-1} = 12$
$2^x \bullet \dfrac{3^x}{3^1} = 12$
$2^x \bullet 3^x = 36$
$6^x = 36$
$6^x = 6^2$
$x = 2$ #

4. Form 6 Question:
If $2^x = 3^y = 6^z$, state the z in terms of x and y.

$2^x = 6^z$.............{1}
$(\times y)$
$2^{xy} = 6^{yz}$..........{2}

$3^y = 6^z$.............{3}
$(\times x)$
$3^{xy} = 6^{xz}$..........{4}

{2}×{4},
$2^{xy} \times 3^{xy} = 6^{yz} \times 6^{xz}$
$6^{xy} = 6^{yz+xz}$
$xy = yz + xz$
$xy = z(y+x)$
$z = \dfrac{xy}{y+x}$ #

Topic 5 Indices and Logarithm:::::::::: v 5.1 Indices :::::::: v 5.14 Zero Indices

1. $x^0 = 1$
[Except 0 (∞)]

2. $1^x = 1$

Combined the {1} and {2},
Solve the equation $(x^2 - 3x + 1)^{x^2-2x} = 1$

$(x^2 - 3x + 1)^{x^2-2x} = 1$

From the {1},
$x^2 - 2x = 0$
$x(x-2) = 0$
$x = 0 @ 2$

From the {2},
$x^2 - 3x + 1 = 1$
$x^2 - 3x = 0$
$x(x-3) = 0$
$x = 0 @ 3$

$\therefore x = 0 @ 2 @ 3$ #

Indices and Logarithm

Topic 5 Indices and Logarithm:::::::::: v 5.1 Indices ::::::::: v 5.16 Fractional Indices

1. $\sqrt[2]{3^1} = 3^{\frac{1}{2}}$

2. $\sqrt[4]{17^3} = 17^{\frac{3}{4}}$

Topic 5 Indices and Logarithm:::::::::: v 5.1 Indices :::::::: v 5.17 Indices Table

Index of 2:	*Index of* 3:	*Index of* 4:	*Index of* 5:	*Index of* 6:	*Index of* 7:
$2^0 = 1$	$3^0 = 1$	$4^0 = 1$	$5^0 = 1$	$6^0 = 1$	$7^0 = 1$
$2^1 = 2$	$3^1 = 3$	$4^1 = 4$	$5^1 = 5$	$6^1 = 6$	$7^1 = 7$
$2^2 = 4$	$3^2 = 9$	$4^2 = 16$	$5^2 = 25$	$6^2 = 36$	$7^2 = 49$
$2^3 = 8$	$3^3 = 27$	$4^3 = 64$	$5^3 = 125$	$6^3 = 216$	$7^3 = 343$
$2^4 = 16$	$3^4 = 81$	$4^4 = 256$	$5^4 = 625$		
$2^5 = 32$	$3^5 = 243$		$5^5 = 3125$		
$2^6 = 64$	$3^6 = 729$				
$2^7 = 128$					
$2^8 = 256$					

Indices and Logarithm

Topic 5 Indices and Logarithm:::::::::: v 5.2 Logarithms :::::: v 5.21 Definition Of Logarithms

e.g.1:

Solve the equation $2^x = 7$.

Method 1:

$$2^x = 7$$
$$\log_{10} 2^x = \log_{10} 7$$
$$\log_{10}(2^x) = \log_{10}(7)$$
$$x\log_{10}(2) = \log_{10}(7)$$
$$x = \frac{\log_{10}(7)}{\log_{10}(2)}$$
$$x = 2.807(3d.p)$$

Method 2:

$$2^x = 7$$
$$\log_e 2^x = \log_e 7$$
$$x\log_e 2 = \log_e 7$$
$$x = \frac{\log_e 7}{\log_e 2}$$
$$x = 2.807(3d.p.)$$

$\log_e \Rightarrow \ln$
Constants:
$e = 2.718281828$
$\pi = 3.141592654$

If $N = a^x$, then $\log_a N = x$

$$N = a^x$$
$$\log_a N = \log_a a^x$$
$$\log_a N = x\log_a a$$
$$\log_a N = x(1)$$
$$\log_a N = x$$
$$\boxed{x = \log_a N\ \#}$$

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e.g.2:

Find the value of x if $3^x = 8$.

$$3^x = 8$$
$$\log_{10} 3^x = \log_{10} 8$$
$$x\log_{10} 3 = \log_{10} 8$$
$$x = \frac{\log_{10} 8}{\log_{10} 3}$$
$$x = 1.893(3d.p.)$$

Topic 5 Indices and Logarithm:::::::::: v 5.2 Logarithms :::::: v 5.22 Definition Of Inverse Logarithms

If $\log_a N = x$, then $N = a^x$

e.g1:

$$\log_2 x = 3$$
$$x = 2^3$$
$$x = 8\ \#$$

e.g.2:

$$\log_{10} x = 0.31$$
$$x = 10^{0.31}$$

$$\log_{10} x = 0.31$$
$$x = \text{antilog } 0.31$$

$$\therefore 10^{0.31} = \text{antilog } 0.31 = 2.042(3d.p.)\ \#$$

Indices and Logarithm

Topic 5 Indices and Logarithm:::::::::: v 5.2 Logarithms ::::::: ***v 5.23 Laws Of Logarithms

**a) $\log xy = \log x + \log y$

**b) $\log\left(\frac{x}{y}\right) = \log x - \log y$

c) $\log x^2 = 2\log x$

d) $\log_x x = 1$

e) $\log_y x\,(x>0, y>0)$

***Important Notes:

$\log x^2 \neq (\log x)^2$

$\log\left(\frac{x}{y}\right) \neq \frac{\log x}{\log y}$

e.g.1:

Simplify $\log\frac{1}{2} + \log\frac{3}{4} - \log\frac{5}{6} - \log\frac{7}{8}$.

Solution:

$$\log\frac{1}{2} + \log\frac{3}{4} - \log\frac{5}{6} - \log\frac{7}{8}$$

$$= \log\left[\frac{1\times 3\times 6\times 8}{2\times 4\times 5\times 7}\right]$$

$$= \log\left(\frac{18}{35}\right) \#$$

e.g.2:

Solve $\log_5 x + \log_5(2x-3) = 1$.

Solution:

$$\log_5 x + \log_5(2x-3) = 1 \Leftarrow \text{Original Equation}$$

$$\log_5\left[x(2x-3)\right] = 1$$

$$2x^2 - 3x = 5^1$$

$$2x^2 - 3x - 5 = 0$$

$$(2x-5)(x+1) = 0$$

$$x = \frac{5}{2} @ -1$$

But $x \neq -1$, then $x = \frac{5}{2}$ #

** Check back the answer from the original equation.

e.g.3:

Solve $\log 5 + \log(2y-1) = \log 2 + \log(y+2)$.

Solution:

$$\log 5 + \log(2y-1) = \log 2 + \log(y+2)$$

$$\log\left[5(2y-1)\right] = \log\left[2(y+2)\right]$$

$$5(2y-1) = 2(y+2)$$

$$10y - 5 = 2y + 4$$

$$8y = 9$$

$$y = \frac{9}{8} \#$$

Indices and Logarithm

Topic 5 Indices and Logarithm:::::::::: v 5.2 Logarithms :::::: v 5.24 Change Of Base Of Logarithms

(a) $\log_y x = \dfrac{\log_a x}{\log_a y}$

e.g.: $\log_2 3 = \dfrac{\log_{10} 3}{\log_{10} 2}$

$= 1.585 (3 d.p.)$

(b) $\log_y x = \dfrac{\log_x x}{\log_x y} = \dfrac{1}{\log_x y}$

e.g.1:

Solve $\frac{1}{16}\log_5 x = \log_x 5$

Solution:

$\frac{1}{16}\log_5 x = \log_x 5$

$\frac{1}{16}\log_5 x = \dfrac{1}{\log_5 x}$

$\frac{1}{16}(\log_5 x)^2 = 1$

$(\log_5 x)^2 = 16$

$\log_5 x = \pm 4$

$x = 5^4 @ 5^{-4}$

$x = 625 @ \dfrac{1}{625}$ #

$\log_2 8 = \log_2 (2^3)$

$= \log_2 (2^3)$

$= 3\log_2 2$

$= 3(1)$

$= 3$#

e.g.2:

Solve $\log_3 x + 2\log_x 3 = 3$

Solution:

$\log_3 x + 2\log_x 3 = 3$

$\log_3 x + 2\left[\dfrac{1}{\log_3 x}\right] = 3$

$\log_3 x + \left[\dfrac{2}{\log_3 x}\right] = 3$

$(\log_3 x)^2 + 2 = 3\log_3 x$

$(\log_3 x)^2 - 3\log_3 x + 2 = 0$

$(\log_3 x - 2)(\log_3 x - 1) = 0$

$\log_3 x = 1 @ 2$

$\log_3 x = 1$ ¦ $\log_3 x = 2$

$x = 3$# ¦ $x = 9$#

Indices and Logarithm

6 Types Of Question: 1) Definition Of Logarithms

Solve the following equation:

1) $3^x = 10$

2) $5^{x-1} = 2$

3) $\frac{1}{2} \cdot 5^{x+1} = 3$

4) $3^{x+2} = 21$

5) $2^x \cdot 3^{x-1} = 6$

6) $y = px^n + 1$, given that $y = 13, x = 2$ and $y = 49, x = 4$, find n.

1) $3^x = 10$

Solution:

$$3^x = 10$$
$$\log_{10} 3^x = \log_{10} 10$$
$$\log_{10} 3^x = 1$$
$$x \log_{10} 3 = 1$$
$$x = \frac{1}{\log_{10} 3}$$
$$x = 2.096 (3d.p.)$$

4) $3^{x+2} = 21$

Solution:

$$3^x \bullet 3^2 = 21$$
$$3^x \bullet 9 = 21$$
$$3^x = \frac{7}{3}$$
$$\log(3^x) = \log\left(\frac{7}{3}\right)$$
$$x \log 3 = \log\left(\frac{7}{3}\right)$$
$$x = \frac{\log\left(\frac{7}{3}\right)}{\log 3}$$
$$x = 0.771243749$$
$$x = 0.7712 \#$$

2) $5^{x-1} = 2$

Solution:

$$5^{x-1} = 2$$
$$\frac{5^x}{5^1} = 2$$
$$5^x = 10$$
$$\log_{10}(5^x) = \log_{10} 10$$
$$x \log_{10} 5 = 1$$
$$x = \frac{1}{\log_{10} 5}$$
$$x = 1.431 (3d.p.)$$

5) $2^x \cdot 3^{x-1} = 6$

Solution:

$$2^x \bullet \frac{3^x}{3^1} = 6$$
$$2^x \bullet 3^x = 6$$
$$6^x = 18$$
$$\log 6^x = \log 18$$
$$x \log 6 = \log 18$$
$$x = \frac{\log 18}{\log 6}$$
$$x = 1.613 (3d.p.) \#$$

3) $\frac{1}{2} \bullet 5^{x+1} = 3$

Solution:

$$\frac{1}{2} \bullet 5^{x+1} = 3$$
$$5^{x+1} = 6$$
$$5^x \cdot 5^1 = 6$$
$$5^x = \frac{6}{5}$$
$$\log_{10}(5^x) = \log_{10}\left(\frac{6}{5}\right)$$
$$x \log_{10} 5 = \log_{10} 6 - \log_{10} 5$$
$$x = \frac{\log_{10} 6 - \log_{10} 5}{\log_{10} 5}$$
$$x = 0.1133 (4d.p.)$$

6) $y = px^n + 1; x = 2, y = 13; x = 4, y = 49$

Solution:

$13 = p(2)^n + 1$

$p(2)^n = 12 \ldots\ldots\ldots\ldots \{1\}$

$49 = p(4)^n + 1$

$p(4)^n = 48 \ldots\ldots\ldots\ldots \{2\}$

$\{2\} \div \{1\}$,

$$\frac{48}{12} = \frac{p(4^n)}{p(2^n)}$$
$$4 = (2^n)$$
$$\log 4 = \log(2^n)$$
$$\log 4 = n \log 2$$
$$\frac{\log 4}{\log 2} = n$$
$$n = 2 \#$$

6 Types Of Question: 3) Given Information

e.g.1:

Given that $\log_2 3 = 1.585$, $\log_2 5 = 2.322$, find $\log_2 30, \log_2 1.2$ and $\log_2 90$.

Solution:

$\log_2 3 = 1.585$

$\log_2 5 = 2.322$

(*i*) $\log_2 30 = \log_2(5 \times 3 \times 2)$
$$= \log_2 5 + \log_2 3 + \log_2 2$$
$$= 2.322 + 1.585 + 1$$
$$= 4.907 (3d.p.) \#$$

(*ii*) $\log_2 1.2 = \log_2\left(\frac{6}{5}\right)$
$$= \log_2 6 - \log_2 5$$
$$= \log_2(2 \times 3) - \log_2 5$$
$$= \log_2 2 + \log_2 3 - \log_2 5$$
$$= 1 + 1.585 - 2.322$$
$$= 0.263 (3d.p.)$$

(*iii*) $\log_2 90 = \log_2(3^2 \times 2 \times 5)$
$$= \log_2 3^2 + \log_2 2 + \log_2 5$$
$$= 2\log_2 3 + \log_2 2 + \log_2 5$$
$$= 2(1.585) + 1 + 2.322$$
$$= 6.492 (3d.p.)$$

Indices and Logarithm

e.g.2:

Given that $\log_y 3 = p, \log_y 5 = q$, find $\log_{15} y$ and $\log_y 15y^2$.

Solution:

$\log_y 3 = p$

$\log_y 5 = q$

(i) $\log_{15} y = \dfrac{1}{\log_y 15}$

$= \dfrac{1}{\log_y (3\times 5)}$

$= \dfrac{1}{\log_y 3 + \log_y 5}$

$= \dfrac{1}{p+q}$ #

(ii) $\log_y 15y^2 = \log_y \left(3\times 5\times y^2\right)$

$= \log_y 3 + \log_y 5 + \log_y y^2$

$= p + q + 2\left(\log_y y\right)$

$= p + q + 2(1)$

$= p + q + 2$ #

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e.g.3:

Given that $\log_x a = 0.63, \log_x b = 1.47$, find $\log_x \sqrt{ab}$ and $\log_{ab} x$.

Solution:

$\log_x a = 0.63$

$\log_x b = 1.47$

(i) $\log_x \sqrt{ab} = \log_x (ab)^{\frac{1}{2}}$

$= \log_x \left(a^{\frac{1}{2}} b^{\frac{1}{2}}\right)$

$= \log_x a^{\frac{1}{2}} + \log_x b^{\frac{1}{2}}$

$= \frac{1}{2}\log_x a + \frac{1}{2}\log_x b$

$= \frac{1}{2}(0.63) + \frac{1}{2}(1.47)$

$= 1.05$ #

(ii) $\log_{ab} x = \dfrac{1}{\log_x (ab)}$

$= \dfrac{1}{\log_x a + \log_x b}$

$= \dfrac{1}{0.63 + 1.47}$

$= \dfrac{10}{21}$ #

*****e.g.4:

Given that $\log_{27} x = a, \log_9 y = b$, find $\log_3 \left(\dfrac{x}{y}\right)$.

Solution:

$\log_{27} x = a$

$\log_9 y = b$

$\log_{27} x = a$

$\dfrac{\log_3 x}{\log_3 27} = a$

$\dfrac{\log_3 x}{3} = a$

$\boxed{\log_3 x = 3a}$

$\log_9 y = b$

$\dfrac{\log_3 y}{\log_3 9} = b$

$\dfrac{\log_3 y}{2} = a$

$\boxed{\log_3 y = 2a}$

$\log_3 \left(\dfrac{x}{y}\right) = \log_3 x - \log_3 y$

$= 3a - 2b$ #

Indices and Logarithm

6 Types Of Question: 4) Change of base

1.) $\log_5 8 + 3\log_3 4 = \dfrac{\log_{10} 8}{\log_{10} 5} + 3\left[\dfrac{\log_{10} 4}{\log_{10} 3}\right]$

$= 5.078\#$

2.) $\log_5 x + 1 = 2\log_x 5$

$\boxed{\log_5 x = \dfrac{1}{\log_x 5}}$

$\log_5 x + 1 = \dfrac{2}{\log_5 x}$

$(\log_5 x)^2 + \log_5 x = 2$

$(\log_5 x)^2 + \log_5 x - 2 = 0$

$(\log_5 x + 2)(\log_5 x - 1) = 0$

$\log_5 x = -2 @ 1$

$x = 5^{-2} @ 5^1$

$x = 0.04 @ 5$

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e.g.2:

If $\log_a xy = 2, \log_a x^3y^2 = 3$, calculate $\log_a x$ and $\log_a y$. Hence, find the value of x and y when $a = 2$.

Solution:

$\log_a xy = 2$

$\log_a x + \log_a y = 2 \ldots\ldots\ldots\ldots\{1\}$

$\log_a x^3y^2 = 3$

$\log_a x^3 + \log_a y^2 = 2$

$3\log_a x + 2\log_a y = 2 \ldots\ldots\ldots\ldots\{2\}$

From $\{1\}$,

$\log_a x + \log_a y = 2$

$(\times 2)$

$2\log_a x + 2\log_a y = 4 \ldots\ldots\ldots\ldots\{3\}$

$\{2\} - \{3\}$,

$\log_a x = -1$

From $\{1\}$,

$(-1) + \log_a y = 2$

$\log_a y = 3$

when $a = 2, \log_2 y = 3$

$y = 2^3$

$y = 8\#$

when $a = 2, \log_2 x = -1$

$x = 2^{-1}$

$x = \dfrac{1}{2}\#$

Indices and Logarithm

6 Types Of Question: 6) Other Questions

1. $\log_x 16 = 8$

$$16 = x^8$$
$$x^8 = 16$$
$$\log_{10} x^8 = \log_{10} 16$$
$$8\log_{10} x = \log_{10} 16$$
$$8\log_{10} x = \log_{10} 16$$
$$\log_{10} x = \frac{\log_{10} 16}{8}$$
$$x = \text{antilog}_{10}\left(\frac{\log_{10} 16}{8}\right)$$
$$x = 1.414(3d.p.)\#$$

2. $\log_{10}\left[x^2 + 6x + 28\right] = 2$

$$\left[x^2 + 6x + 28\right] = 10^2$$
$$x^2 + 6x + 28 = 100$$
$$x^2 + 6x - 72 = 0$$
$$(x+12)(x-6) = 0$$
$$x = -12 @ 6\#$$

3. Given that $2\log_3\left(xy^2\right) = 3 - \log_3\left(\frac{x}{y^7}\right)$. Express the x in terms of y. If $y^2 - 2x + 5 = 0$,

Find the value of x and y.

$$2\log_3\left(xy^2\right) = 3 - \log_3\left(\frac{x}{y^7}\right)$$
$$\log_3\left(xy^2\right)^2 + \log_3\left(\frac{x}{y^7}\right) = 3$$
$$\log_3\left(x^2y^4\right) + \log_3\left(\frac{x}{y^7}\right) = 3$$
$$\log_3 x^2 + \log_3 y^4 + \log_3 x - \log_3 y^7 = 3$$
$$2\log_3 x + 4\log_3 y + \log_3 x - 7\log_3 y = 3$$
$$3\log_3 x - 3\log_3 y = 3$$
$$3\left(\log_3 x - \log_3 y\right) = 3$$
$$\log_3 x - \log_3 y = 1$$
$$\log_3\left(\frac{x}{y}\right) = 1$$
$$\left(\frac{x}{y}\right) = 3^1$$
$$x = 3y \text{.............}\{1\}$$

$$y^2 - 2x + 5 = 0 \text{.............}\{2\}$$

Subtitute x into $\{2\}$,

$$y^2 - 2(3y) + 5 = 0$$
$$y^2 - 6y + 5 = 0$$
$$(y-1)(y-5) = 0$$
$$y = 1 @ 5$$

From $\{1\}$,

when $y = 1, x = 3(1) = 3$ | when $y = 5, x = 3(5) = 15$

$$x = 3, y = 1; x = 15, y = 5\#$$

***4.)

Given that the total of saving money, F for a total money that saved in bank is given by the equation $F = P\left(1+\frac{r}{100}\right)^n$ where P is a derivation of saving money, r is a rate of annual interest and n is the number of year the money saved in bank. If Ali was saved RM 1200 with the rate of 3.5% per year, find the period for the total of saving money is become RM 2000.

$$F = P\left(1+\frac{r}{100}\right)^n$$

$$1200\left(1+\frac{3.5}{100}\right)^n = 2000$$

$$(1.035)^n = \frac{5}{3}$$

$$\log_{10}\left[(1.035)^n\right] = \log_{10}\left(\frac{5}{3}\right)$$

$$n\log_{10}(1.035) = \log_{10}\left(\frac{5}{3}\right)$$

$$n\log_{10}(1.035) = \log_{10}\left(\frac{5}{3}\right) \div \log_{10}(1.035)$$

$$n = \log_{10}\left(\frac{5}{3}\right) \times \frac{1}{\log_{10}(1.035)}$$

$$n = 14.85(2d.p.)$$

$$n \approx 15 \text{ years\#}$$

e.g.:

$F = 50 \times 1.10$

$F = 50\left(1+\frac{10}{100}\right)^1$

$F = 55\#$

Exercise 2:

1. If $\log_a 3 = m$, express $\log_a\left(\frac{81}{a^2}\right)$ in terms of m.
2. Without using tables or a calculator, find the value of $5^{\frac{1}{2}\log_5 9}$.
3. Find the value of $\log_b \sqrt{b} - \log_a\left(\frac{1}{a}\right)$.
4. Given that $a = 3^p$, and $b = 3^q$, find $\log_9 a + \log_{27} b$ in terms of p and q.
5. Solve each of the following equation:
   - (a) $3^{m-1} + 3^m - 12 = 0$
   - (b) $4^t = 23.4$
   - (c) $2^{x+3} - 7 = 0$
   - (d) $3^x \bullet 5^x = 8^{x+1}$
   - (e) $\log_t(t-2) + \log_t(t+5) = 2$
   - (f) $2 - \log y = 3\log x$, express y in terms of x.
   - (g) $\log_{49}\left[\log_2(5x-2)\right] = \frac{1}{2}$
   - (h) $\log_2 x - \log_4 x = -2$
6. (a) Given that $2\log_2(x+y) = 3 + \log_2 x + \log_2 y$, show that $x^2 + y^2 = 6xy$.

   (b) Without using tables or a calculator, solve the equation $\log_9\left[\log_3(3x-6)\right] = 5^{\log_5\left(\frac{1}{2}\right)}$.

   (c) The value of machine (in RM). after n years is given by $45000\left(\frac{23}{25}\right)^n$. Calculate the minimum number of years for the value of the machine to be less than RM 20000 for the first time.

END OF TOPIC 5(Indices And Logarithms)