Add Maths Formulae List: Form 4 (Update 20/07/2010)

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01 Functions

Absolute Value Function	**Inverse Function**
$f(x) \begin{cases} f(x), & \text{if } f(x) \geq 0 \\ -f(x), & \text{if } f(x) < 0 \end{cases}$	If $y = f(x)$, then $f^{-1}(y) = x$ Remember: Object = the value of x Image = the value of y or $f(x)$ $f(x)$ map onto itself means $f(x) = x$

02 Quadratic Equations

General Form	**Quadratic Formula**
$\boxed{ax^2 + bx + c = 0}$ where a, b, and c are constants and $a \neq 0$. *Note that the highest power of an unknown of a quadratic equation is **2**.	$\boxed{x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$ When the equation can not be factorized.
Forming Quadratic Equation From its Roots: If α and β are the roots of a quadratic equation $\boxed{\alpha + \beta = -\dfrac{b}{a}}$ $\boxed{\alpha\beta = \dfrac{c}{a}}$ The Quadratic Equation $\boxed{x^2 - (\alpha + \beta)x + \alpha\beta = 0}$ or $\boxed{x^2 - (SoR)x + (PoR) = 0}$ SoR = Sum of Roots PoR = Product of Roots	**Nature of Roots** $b^2 - 4ac > 0 \Leftrightarrow$ two real and different roots $b^2 - 4ac = 0 \Leftrightarrow$ two real and equal roots $b^2 - 4ac < 0 \Leftrightarrow$ no real roots $b^2 - 4ac \geq 0 \Leftrightarrow$ the roots are real

03 Quadratic Functions

General Form

$$f(x) = ax^2 + bx + c$$

where a, b, and c are constants and $a \neq 0$.

*Note that the highest power of an unknown of a quadratic function is **2**.

$a > 0 \Rightarrow$ minimum $\Rightarrow$ $\cup$ (smiling face)

$a < 0 \Rightarrow$ maximum $\Rightarrow$ $\cap$ (sad face)

Completing the square:

$$f(x) = a(x + p)^2 + q$$

(i) the value of x, $\boxed{x = -p}$

(ii) min./max. value = q

(iii) min./max. point = $(-p, q)$

(iv) equation of axis of symmetry, $\boxed{x = -p}$

Alternative method:

$$f(x) = ax^2 + bx + c$$

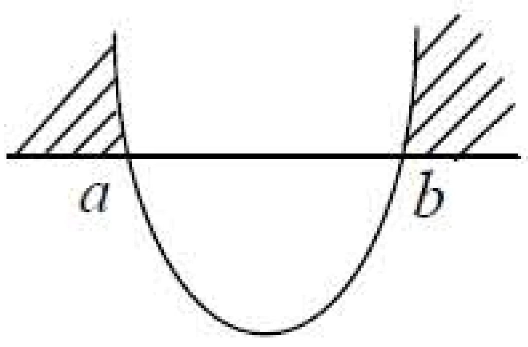
(i) the value of x, $\boxed{x = -\frac{b}{2a}}$

(ii) min./max. value = $f(-\frac{b}{2a})$

(iii) equation of axis of symmetry, $\boxed{x = -\frac{b}{2a}}$

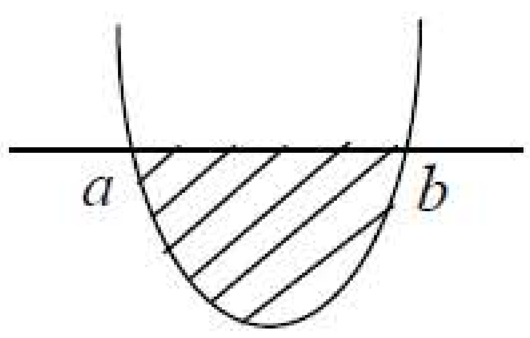
Quadratic Inequalities

$a > 0$ and $f(x) > 0$



$x < a$ or $x > b$

$a > 0$ and $f(x) < 0$



$a < x < b$

Nature of Roots

$b^2 - 4ac > 0$ $\Leftrightarrow$ intersects two different points at x-axis

$b^2 - 4ac = 0$ $\Leftrightarrow$ touch one point at x-axis

$b^2 - 4ac < 0$ $\Leftrightarrow$ does not meet x-axis

04 Simultaneous Equations

To find the intersection point $\Rightarrow$ solves simultaneous equation.

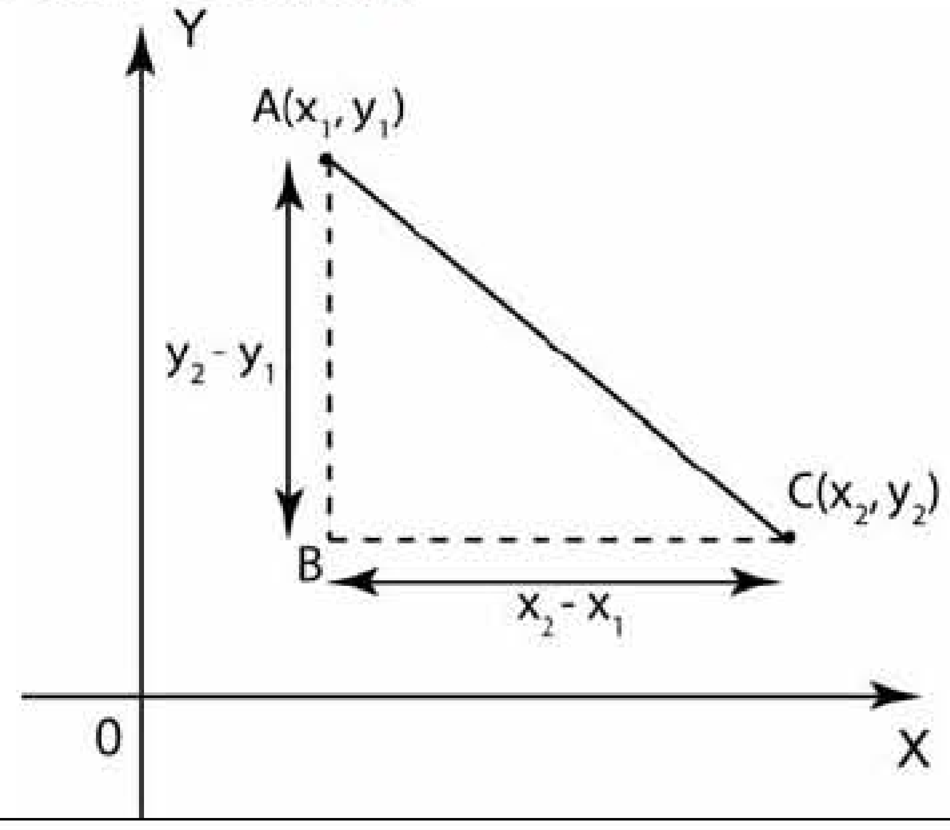
Remember: substitute linear equation into non- linear equation.

05 Indices and Logarithm

Fundamental if Indices	**Laws of Indices**
Zero Index, $a^0 = 1$ Negative Index, $a^{-1} = \frac{1}{a}$ $(\frac{a}{b})^{-1} = \frac{b}{a}$ Fractional Index $a^{\frac{1}{n}} = \sqrt[n]{a}$ $a^{\frac{m}{n}} = \sqrt[n]{a^m}$	$a^m \times a^n = a^{m+n}$ $a^m \div a^n = a^{m-n}$ $(a^m)^n = a^{m \times n}$ $(ab)^n = a^n b^n$ $(\frac{a}{b})^n = \frac{a^n}{b^n}$
Fundamental of Logarithm	**Law of Logarithm**
$\boxed{\log_a y = x \Leftrightarrow a^x = y}$ $\log_a a = 1$ $\log_a a^x = x$ $\log_a 1 = 0$	$\log_a mn = \log_a m + \log_a n$ $\log_a \frac{m}{n} = \log_a m - \log_a n$ $\log_a m^n = n \log_a m$ **Changing the Base** $\log_a b = \frac{\log_c b}{\log_c a}$ $\log_a b = \frac{1}{\log_b a}$

06 Coordinate Geometry

Distance and Gradient



Distance Between Point A and C =

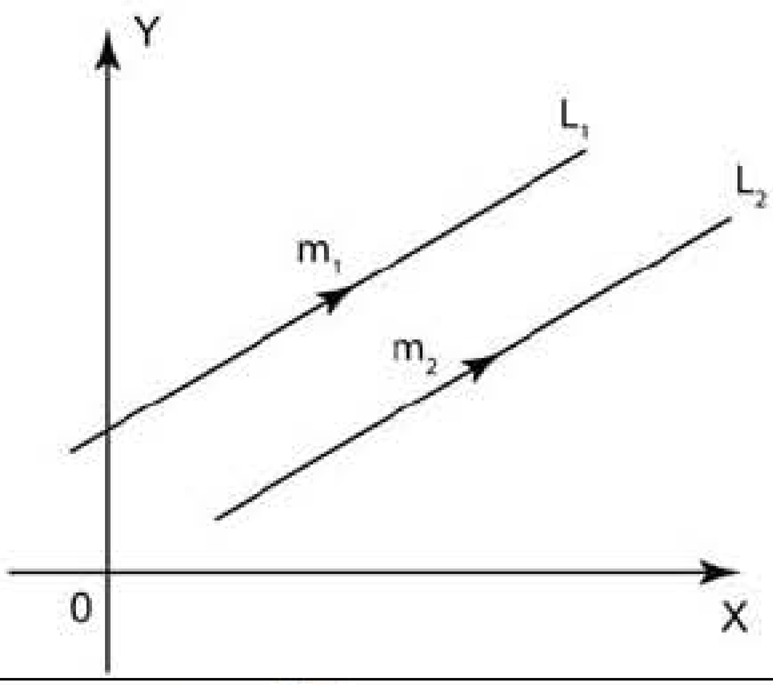
$$\sqrt{(x_1 - x_2)^2 + (x_1 - x_2)^2}$$

Gradient of line AC, $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Or

Gradient of a line, $m = -\left(\dfrac{y - \text{intercept}}{x - \text{intercept}}\right)$

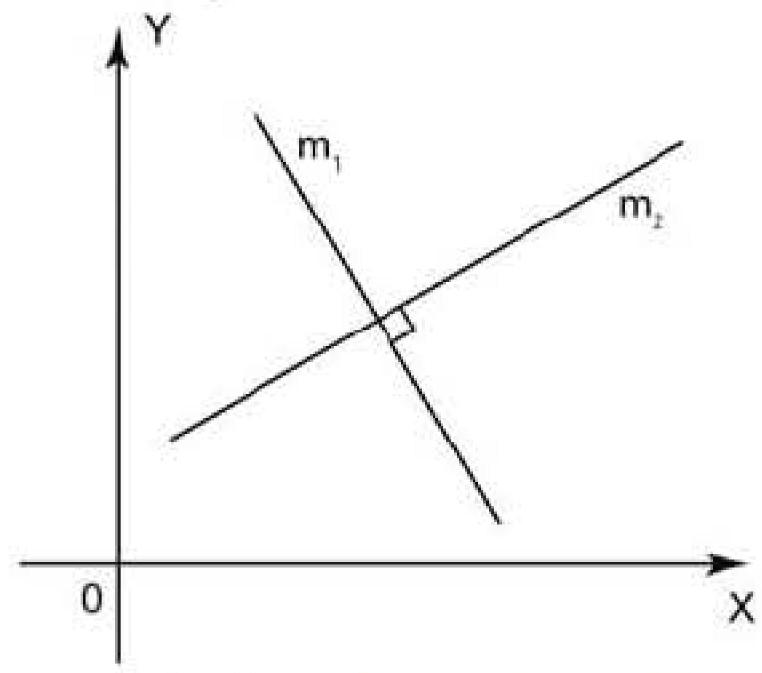
Parallel Lines



When 2 lines are parallel,

$$m_1 = m_2.$$

Perpendicular Lines



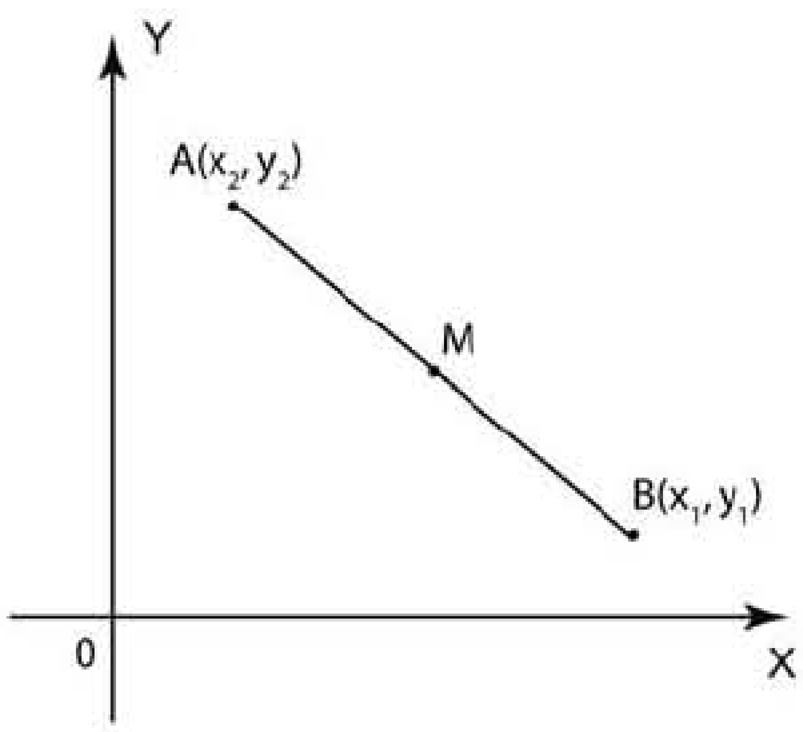
When 2 lines are perpendicular to each other,

$$m_1 \times m_2 = -1$$

m_1 = gradient of line 1

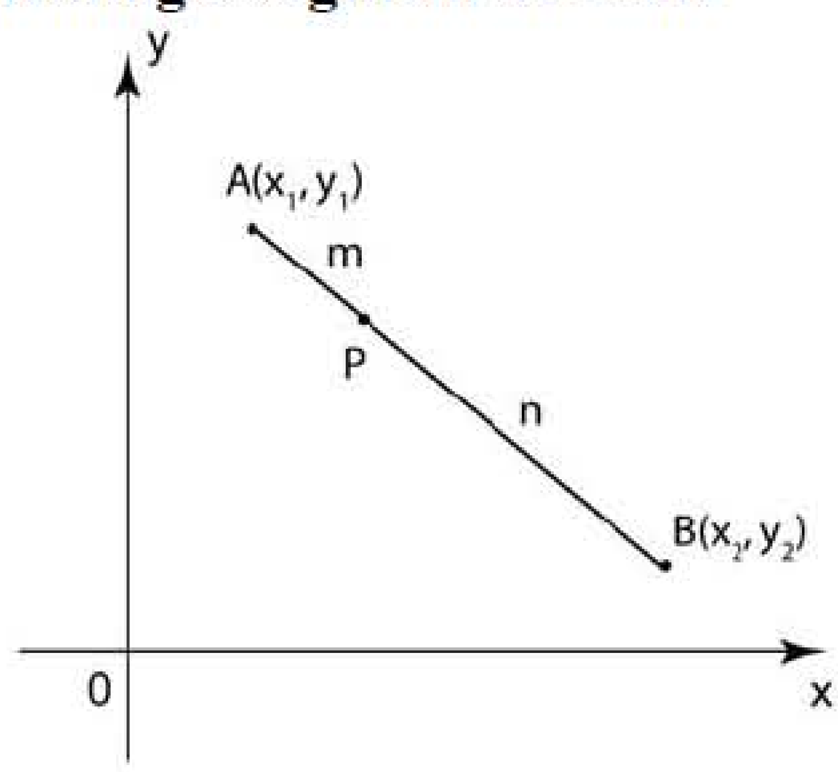
m_2 = gradient of line 2

Midpoint



Midpoint, $M = \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

A point dividing a segment of a line



A point dividing a segment of a line

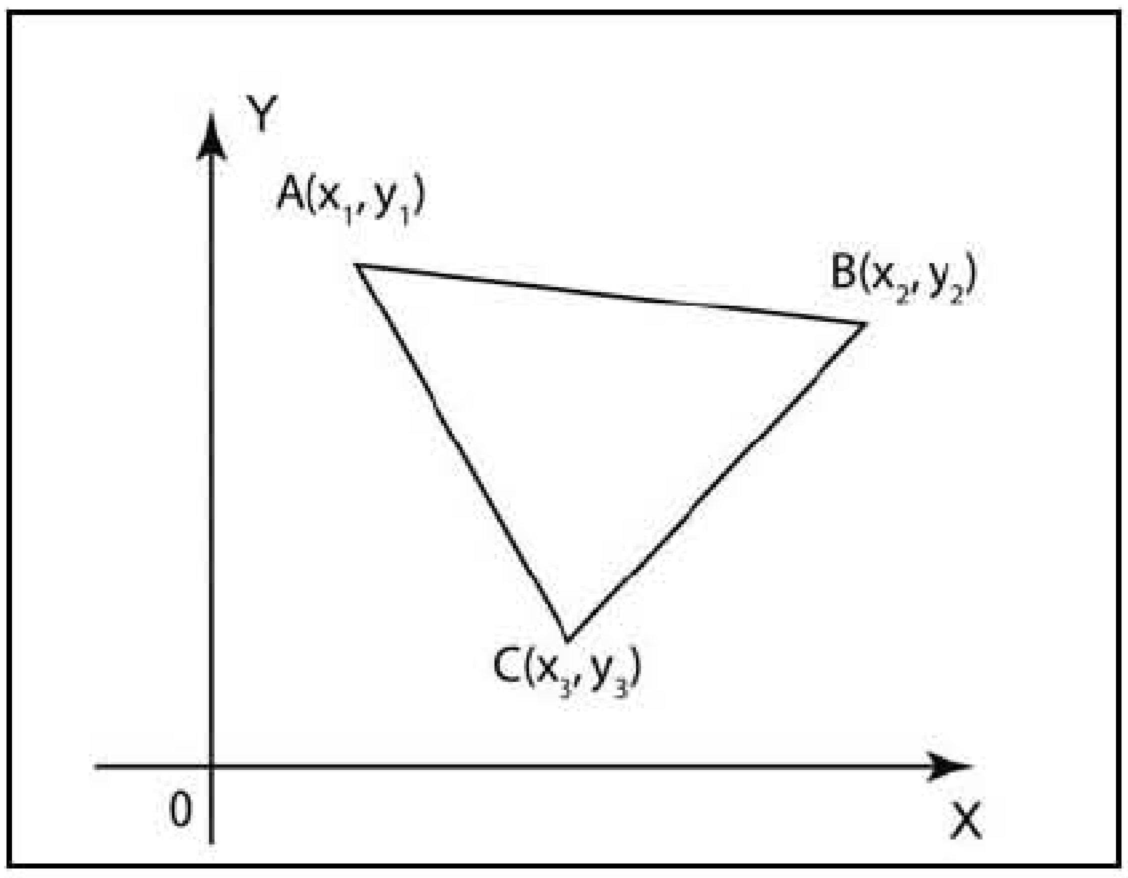
$$P = \left(\frac{nx_1 + mx_2}{m + n}, \frac{ny_1 + my_2}{m + n}\right)$$

Area of triangle:

Area of Triangle

$$= \frac{1}{2}\begin{vmatrix} \square & \square & \square & \square \\ \square & \square & \square & \square \end{vmatrix}$$

$$A = \frac{1}{2}\left|\left(x_1y_2 + x_2y_3 + x_3y_1\right) - \left(x_2y_1 + x_3y_2 + x_1y_3\right)\right|$$



Form of Equation of Straight Line

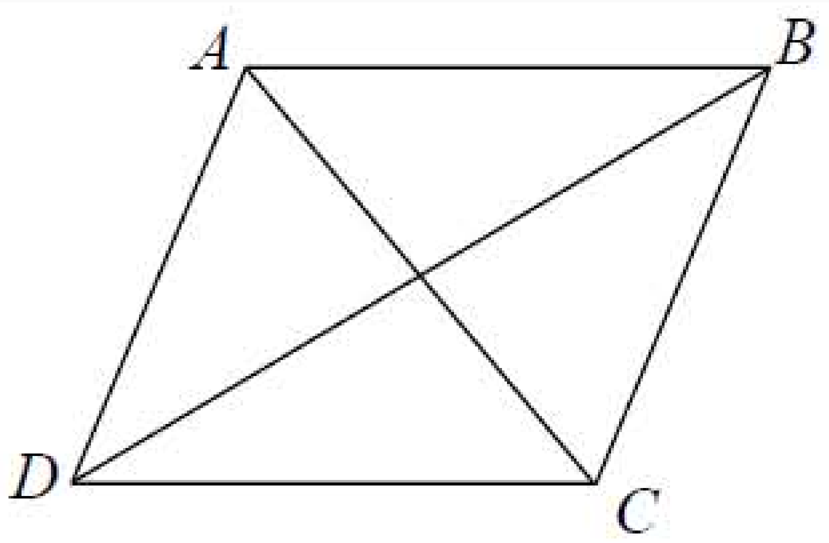
General form	***Gradient form***	***Intercept form***
$ax + by + c = 0$	$y = mx + c$ m = gradient c = y-intercept	$\frac{x}{a} + \frac{y}{b} = 1$ a = x-intercept b = y-intercept $m = -\frac{b}{a}$

Equation of Straight Line

Gradient (m) and 1 point (x_1, y_1) given	2 points, (x_1, y_1) and (x_2, y_2) given	x-intercept and y-intercept given
$y - y_1 = m(x - x_1)$	$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$	$\frac{x}{a} + \frac{y}{b} = 1$

Equation of perpendicular bisector $\Rightarrow$ gets midpoint and gradient of perpendicular line.

Information in a rhombus:



(i) same length $\Rightarrow$ $AB = BC = CD = AD$

(ii) parallel lines $\Rightarrow$ $m_{AB} = m_{CD}$ or $m_{AD} = m_{BC}$

(iii) diagonals (perpendicular) $\Rightarrow$ $m_{AC} \times m_{BD} = -1$

(iv) share same midpoint $\Rightarrow$ midpoint AC = midpoint BD

(v) any point $\Rightarrow$ solve the simultaneous equations

Remember:
y-intercept $\Rightarrow$ $x=0$ cut y-axis $\Rightarrow$ $x=0$ x-intercept $\Rightarrow$ $y=0$ cut x-axis $\Rightarrow$ $y=0$ **point lies on the line $\Rightarrow$ satisfy the equation $\Rightarrow$ substitute the value of x and of y of the point into the equation.

Equation of Locus		
(use the formula of distance) The equation of the locus of a moving point $P(x, y)$ which is always at a constant distance (r) from a fixed point $A(x_1, y_1)$ is $$PA = r$$ $$(x-x_1)^2+(y-y_1)^2=r^2$$	The equation of the locus of a moving point $P(x, y)$ which is always at a constant distance from two fixed points $A(x_1, y_1)$ and $B(x_2, y_2)$ with a ratio $m:n$ is $$\frac{PA}{PB}=\frac{m}{n}$$ $$\frac{(x-x_1)^2+(y-y_1)^2}{(x-x_2)+(y-y_2)^2}=\frac{m^2}{n^2}$$	The equation of the locus of a moving point $P(x, y)$ which is always equidistant from two fixed points A and B is the ***perpendicular bisector*** of the straight line AB. $$PA = PB$$ $$(x-x_1)^2+(y-y_1)^2=(x-x_2)^2+(y-y_2)^2$$

07 Statistics

Measure of Central Tendency

	Ungrouped Data	Grouped Data	
		Without Class Interval	With Class Interval
Mean	$\bar{x} = \frac{\Sigma x}{N}$ $\bar{x}$ = mean Σx = sum of x x = value of the data N = total number of the data	$\bar{x} = \frac{\Sigma fx}{\Sigma f}$ $\bar{x}$ = mean Σx = sum of x f = frequency x = value of the data	$\bar{x} = \frac{\Sigma fx}{\Sigma f}$ $\bar{x}$ = mean f = frequency x = class mark $= \frac{\text{(lower limit+upper limit)}}{2}$
Median	$m = T_{\frac{N+1}{2}}$ When N is an odd number. $m = \frac{T_{\frac{N}{2}} + T_{\frac{N}{2}+1}}{2}$ When N is an even number.	$m = T_{\frac{N+1}{2}}$ When N is an odd number. $m = \frac{T_{\frac{N}{2}} + T_{\frac{N}{2}+1}}{2}$ When N is an even number.	$m = L + \left(\frac{\frac{1}{2}N - F}{f_m}\right)C$ m = median L = Lower boundary of median class N = Number of data F = Total frequency before median class f_m = Total frequency in median class c = Size class = (Upper boundary – lower boundary)

Measure of Dispersion

	Ungrouped Data	Grouped Data	
		Without Class Interval	With Class Interval
variance	$\sigma^2 = \frac{\sum x^2}{N} - \bar{x}^2$	$\sigma^2 = \frac{\sum fx^2}{\sum f} - \bar{x}^2$	$\sigma^2 = \frac{\sum fx^2}{\sum f} - \bar{x}^2$
Standard Deviation	$\sigma = \sqrt{\text{variance}}$ $\sigma = \sqrt{\frac{\Sigma (x - \bar{x})^2}{N}}$ $\sigma = \sqrt{\frac{\Sigma x^2}{N} - \bar{x}^2}$	$\sigma = \sqrt{\text{variance}}$ $\sigma = \sqrt{\frac{\Sigma (x - \bar{x})^2}{N}}$ $\sigma = \sqrt{\frac{\Sigma x^2}{N} - \bar{x}^2}$	$\sigma = \sqrt{\text{variance}}$ $\sigma = \sqrt{\frac{\Sigma f (x - \bar{x})^2}{\Sigma f}}$ $\sigma = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \bar{x}^2}$

The ***variance*** is a measure of the ***mean*** for the square of the ***deviations*** from the ***mean***.

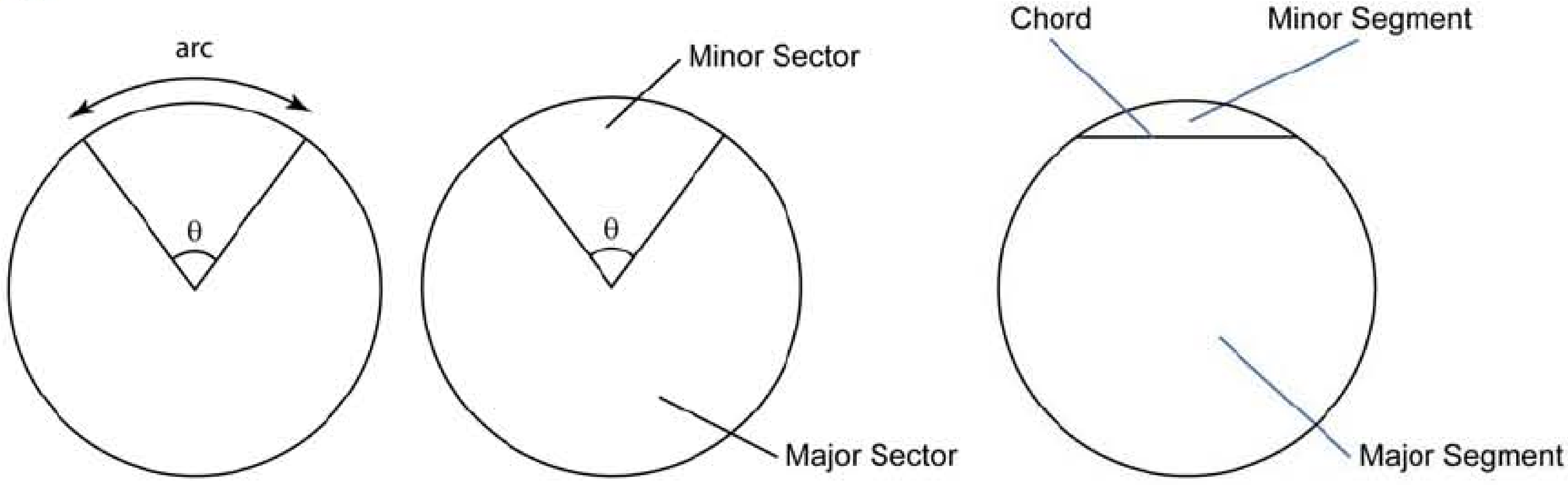
The ***standard deviation*** refers to the ***square root*** for the ***variance***.

Effects of data changes on Measures of Central Tendency and Measures of dispersion

		Data are changed uniformly with			
		$+k$	$-k$	$\times k$	$\div k$
Measures of Central Tendency	Mean, median, mode	$+k$	$-k$	$\times k$	$\div k$
Measures of dispersion	Range , Interquartile Range	No changes		$\times k$	$\div k$
	Standard Deviation	No changes		$\times k$	$\div k$
	Variance	No changes		$\times k^2$	$\div k^2$

08 Circular Measures

Terminology



Convert degree to radian:
Convert radian to degree:

radians $\xrightarrow{\times \frac{180^\circ}{\pi}}$ degrees

radians $\xleftarrow{\times \frac{\pi}{180^\circ}}$ degrees

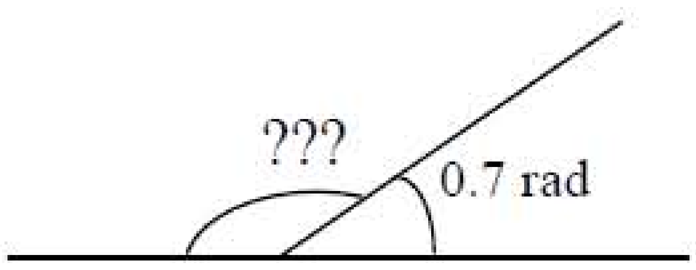
$$x^o = (x \times \frac{\pi}{180})\text{radians}$$

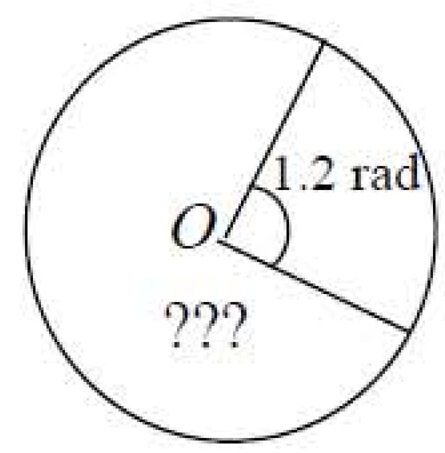
$$x\,\text{radians} = (x \times \frac{180}{\pi})\text{ degrees}$$

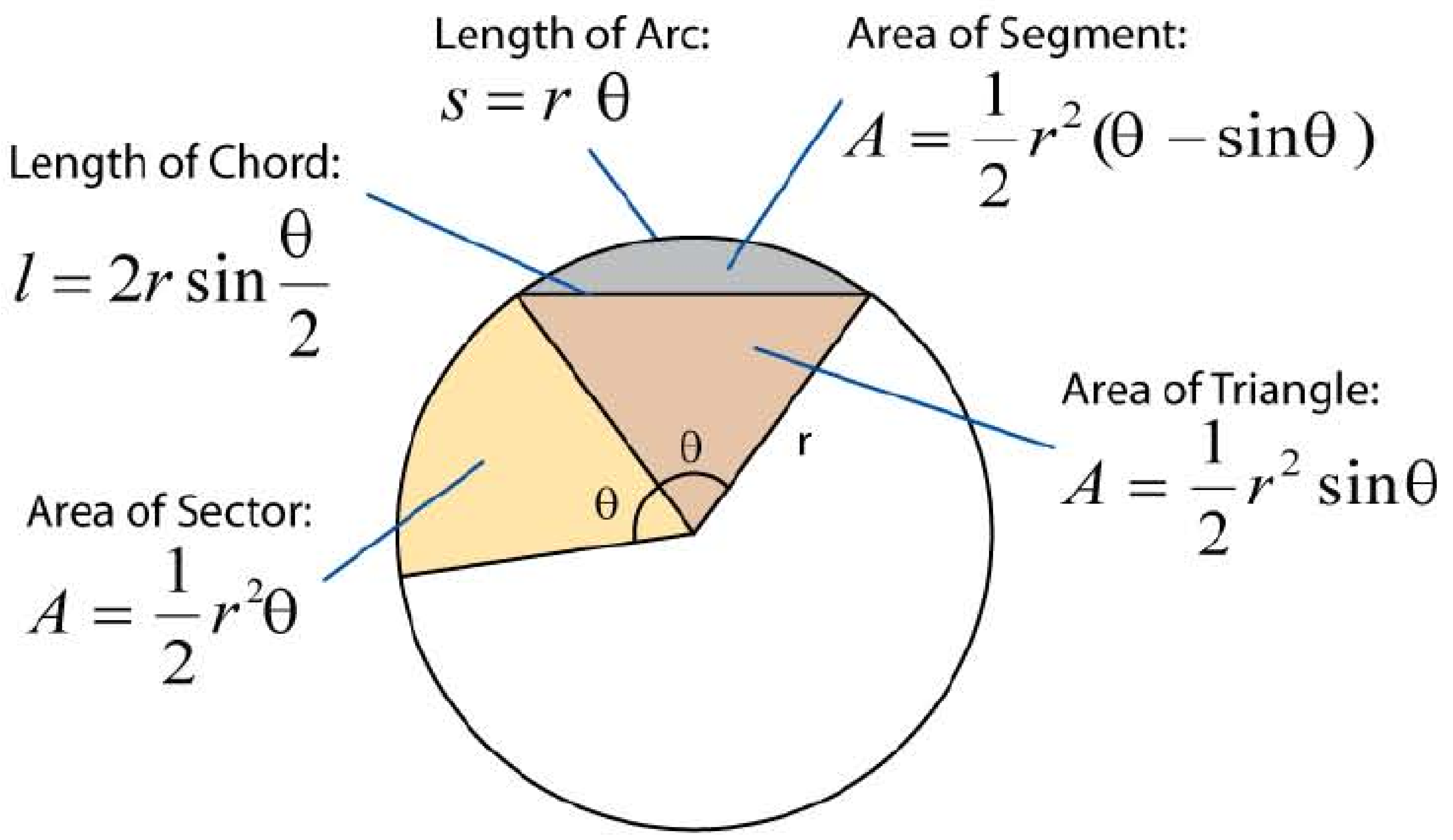
Remember:

$180^\circ = \pi$ rad

$360^\circ = 2\pi$ rad







r = radius
A = area
s = arc length
θ = angle
l = length of chord

Arc Length:	Length of chord:	Area of Sector:	Area of Triangle:	Area of Segment:
$s = r\theta$	$l = 2r\sin\frac{\theta}{2}$	$A = \frac{1}{2}r^2\theta$	$A = \frac{1}{2}r^2\sin\theta$	$A = \frac{1}{2}r^2(\theta - \sin\theta)$

09 Differentiation

Gradient of a tangent of a line (curve or straight)

$$\frac{dy}{dx} = \lim_{\delta x \to 0}\left(\frac{\delta y}{\delta x}\right)$$

Differentiation of Algebraic Function
Differentiation of a Constant

$y = a$ a is a constant

$$\frac{dy}{dx} = 0$$

Example
$y = 2$

$$\frac{dy}{dx} = 0$$

Differentiation of a Function I

$y = x^n$

$$\frac{dy}{dx} = nx^{n-1}$$

Example
$y = x^3$

$$\frac{dy}{dx} = 3x^2$$

Differentiation of a Function II

$y = ax$

$$\frac{dy}{dx} = ax^{1-1} = ax^0 = a$$

Example
$y = 3x$

$$\frac{dy}{dx} = 3$$

Differentiation of a Function III

$y = ax^n$

$\frac{dy}{dx} = anx^{n-1}$

Example

$y = 2x^3$

$\frac{dy}{dx} = 2(3)x^2 = 6x^2$

Differentiation of a Fractional Function

$y = \frac{1}{x^n}$

Rewrite

$y = x^{-n}$

$\frac{dy}{dx} = -nx^{-n-1} = \frac{-n}{x^{n+1}}$

Example

$y = \frac{1}{x}$

$y = x^{-1}$

$\frac{dy}{dx} = -1x^{-2} = \frac{-1}{x^2}$

Law of Differentiation

Sum and Difference Rule

$y = u \pm v$ $\quad$ u and v are functions in x

$\frac{dy}{dx} = \frac{du}{dx} \pm \frac{dv}{dx}$

Example

$y = 2x^3 + 5x^2$

$\frac{dy}{dx} = 2(3)x^2 + 5(2)x = 6x^2 + 10x$

Chain Rule

$y = u^n$ $\quad$ u and v are functions in x

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

Example

$y = (2x^2 + 3)^5$

$u = 2x^2 + 3,$ therefore $\frac{du}{dx} = 4x$

$y = u^5,$ therefore $\frac{dy}{du} = 5u^4$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$= 5u^4 \times 4x$

$= 5(2x^2 + 3)^4 \times 4x = 20x(2x^2 + 3)^4$

Or differentiate directly

$y = (ax + b)^n$

$\frac{dy}{dx} = n.a.(ax + b)^{n-1}$

$y = (2x^2 + 3)^5$

$\frac{dy}{dx} = 5(2x^2 + 3)^4 \times 4x = 20x(2x^2 + 3)^4$

Product Rule

$y = uv$ u and v are functions in x

$$\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$$

Example

$y = (2x+3)(3x^3 - 2x^2 - x)$

$u = 2x+3$ $v = 3x^3 - 2x^2 - x$

$\frac{du}{dx} = 2$ $\frac{dv}{dx} = 9x^2 - 4x - 1$

$$\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$$

$$= (3x^3 - 2x^2 - x)(2) + (2x+3)(9x^2 - 4x - 1)$$

Or differentiate directly

$y = (2x+3)(3x^3 - 2x^2 - x)$

$$\frac{dy}{dx} = (3x^3 - 2x^2 - x)(2) + (2x+3)(9x^2 - 4x - 1)$$

Quotient Rule

$y = \frac{u}{v}$ u and v are functions in x

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Example

$y = \frac{x^2}{2x+1}$

$u = x^2$ $v = 2x+1$

$\frac{du}{dx} = 2x$ $\frac{dv}{dx} = 2$

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$\frac{dy}{dx} = \frac{(2x+1)(2x) - x^2(2)}{(2x+1)^2}$$

$$= \frac{4x^2 + 2x - 2x^2}{(2x+1)^2} = \frac{2x^2 + 2x}{(2x+1)^2}$$

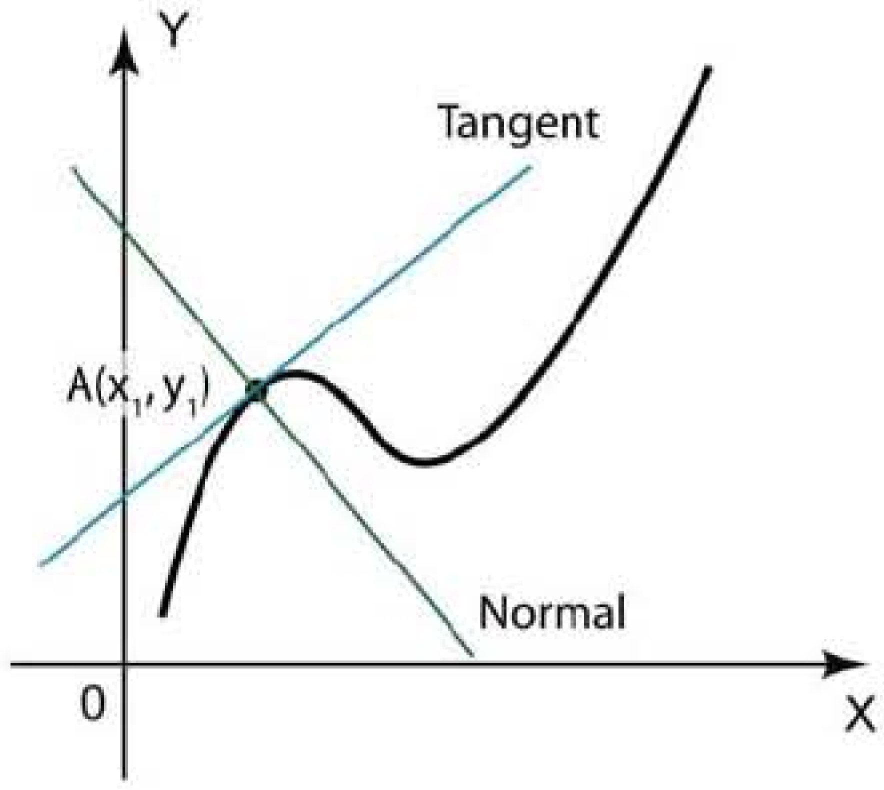
Or differentiate directly

$y = \frac{x^2}{2x+1}$

$$\frac{dy}{dx} = \frac{(2x+1)(2x) - x^2(2)}{(2x+1)^2}$$

$$= \frac{4x^2 + 2x - 2x^2}{(2x+1)^2} = \frac{2x^2 + 2x}{(2x+1)^2}$$

Gradients of tangents, Equation of tangent and Normal



If A(x_1, y_1) is a point on a line $y = f(x)$, the gradient of the line (for a straight line) or the gradient of the tangent of the line (for a curve) is the value of $\frac{dy}{dx}$ when x = x_1.

Gradient of tangent at A(x_1, y_1):

$$\boxed{\frac{dy}{dx} = \text{gradient of tangent}}$$

Equation of tangent: $\boxed{y - y_1 = m(x - x_1)}$

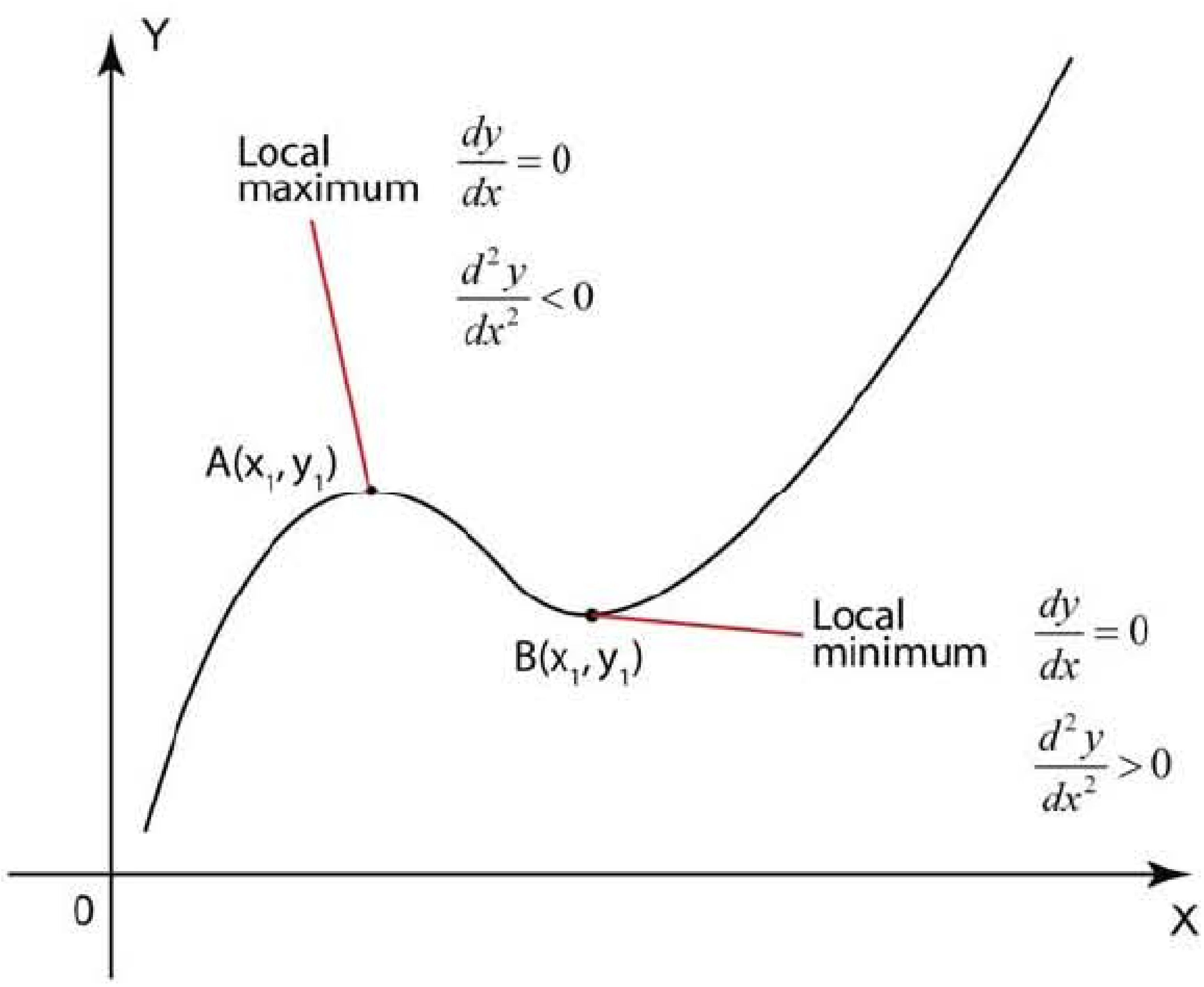
Gradient of normal at A(x_1, y_1):

$$m_{\text{normal}} = -\frac{1}{m_{\text{tangent}}}$$

$$\boxed{\frac{1}{-\frac{dy}{dx}} = \text{gradient of normal}}$$

Equation of normal : $y - y_1 = m(x - x_1)$

Maximum and Minimum Point



Turning point $\Rightarrow$ $\boxed{\frac{dy}{dx} = 0}$

At maximum point,

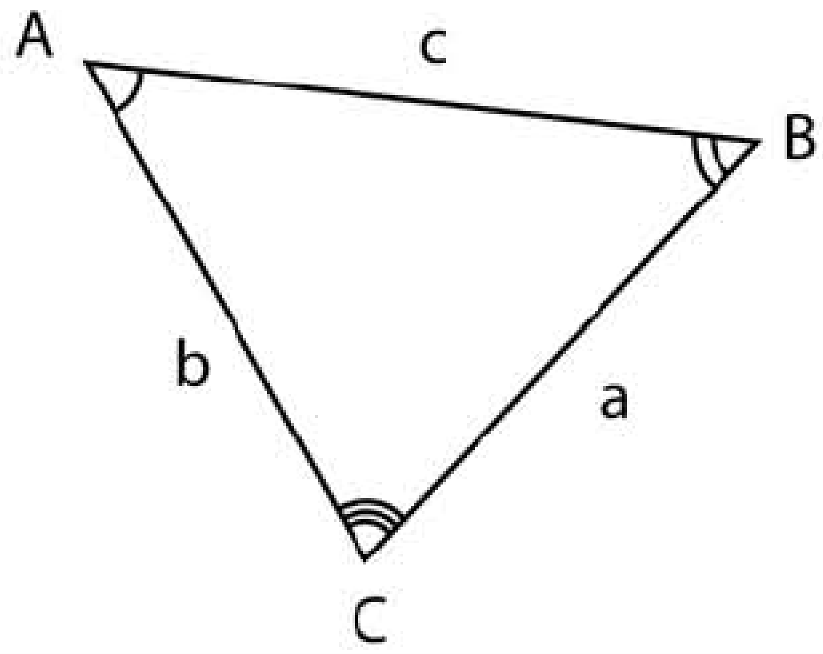
$\boxed{\frac{dy}{dx} = 0}$ $\boxed{\frac{d^2y}{dx^2} < 0}$

At minimum point ,

$\boxed{\frac{dy}{dx} = 0}$ $\boxed{\frac{d^2y}{dx^2} > 0}$

Rates of Change	Small Changes and Approximation
Chain rule $\boxed{\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}}$ If x changes at the rate of 5 cms $^{-1} \Rightarrow \frac{dx}{dt} = 5$ Decreases/leaks/reduces $\Rightarrow$ NEGATIVES values!!!	**Small Change:** $\boxed{\frac{\delta y}{\delta x} \approx \frac{dy}{dx} \Rightarrow \delta y \approx \frac{dy}{dx} \times \delta x}$ **Approximation:** $y_{new} = y_{original} + \delta y$ $= y_{original} + \frac{dy}{dx} \times \delta x$ δx = small changes in x δy = small changes in y If x becomes smaller $\Rightarrow$ $\delta x = NEGATIVE$

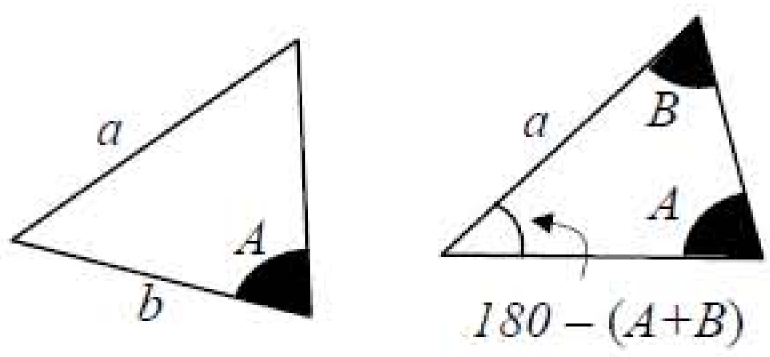
10 Solution of Triangle



Sine Rule:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Use, when given
- 2 sides and 1 non included angle
- 2 angles and 1 side

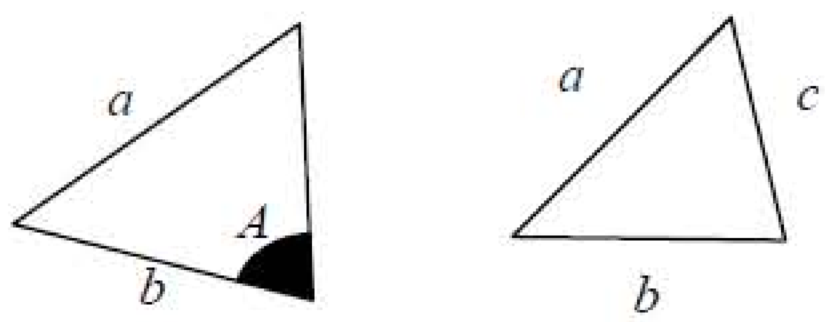


Cosine Rule:

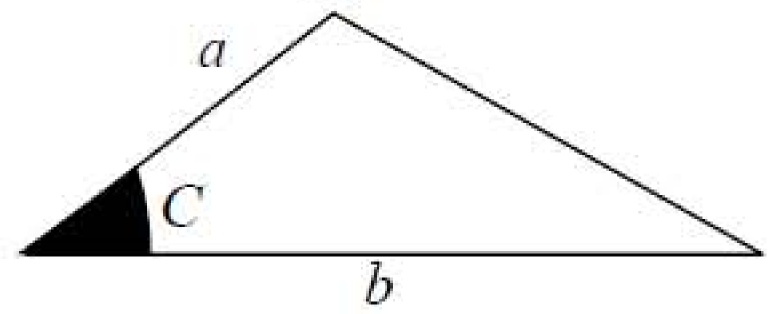
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

Use, when given
- 2 sides and 1 included angle
- 3 sides



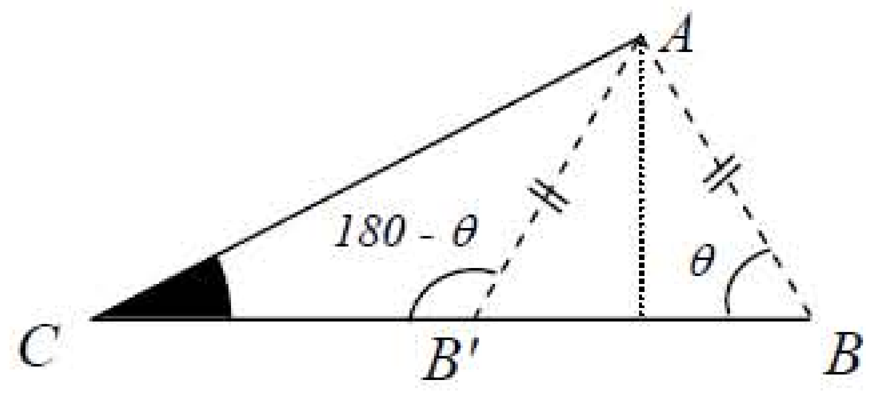
Area of triangle:



$$A = \frac{1}{2}ab\sin C$$

C is the included angle of sides a and b.

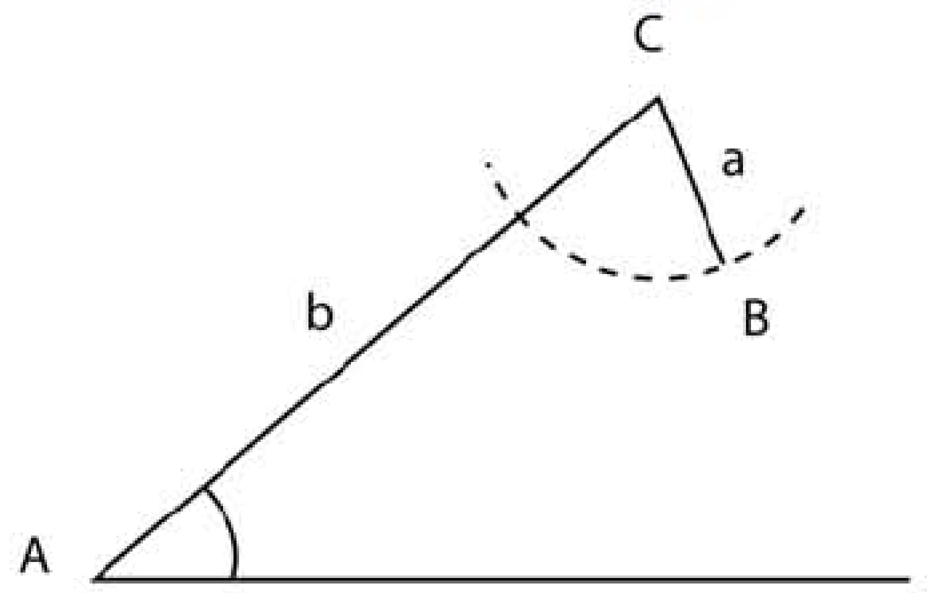
<u>Case of AMBIGUITY</u>



If $\angle C$, the length AC and length AB remain unchanged, the point B can also be at point B' where $\angle ABC$ = acute and $\angle AB'C$ = obtuse.
If $\angle ABC = \theta$, thus $\angle AB'C = 180 - \theta$.

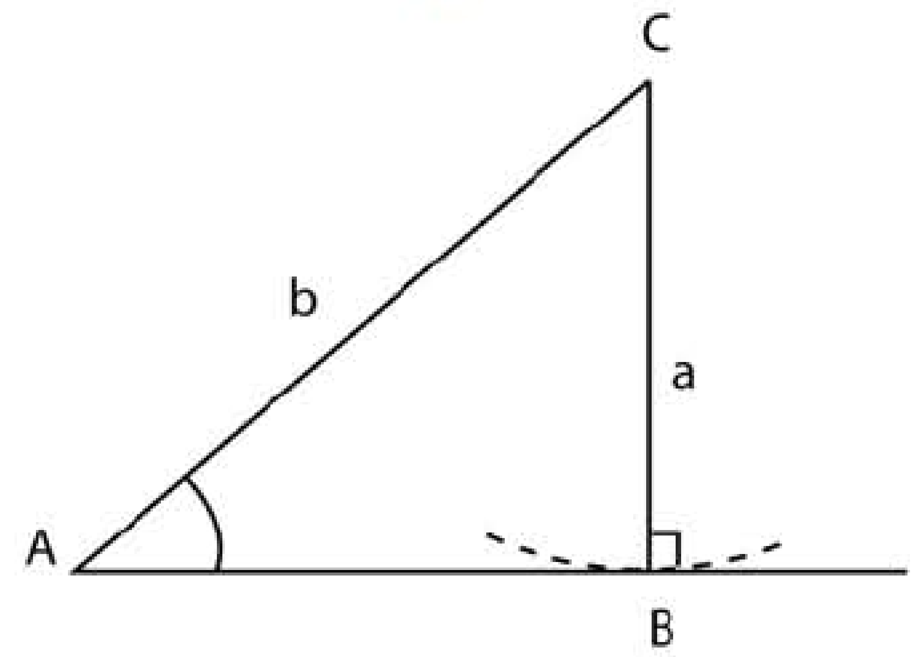
Remember : $\sin\theta = \sin(180° - \theta)$

Case 1: When $a < b\sin A$
CB is too short to reach the side opposite to C.



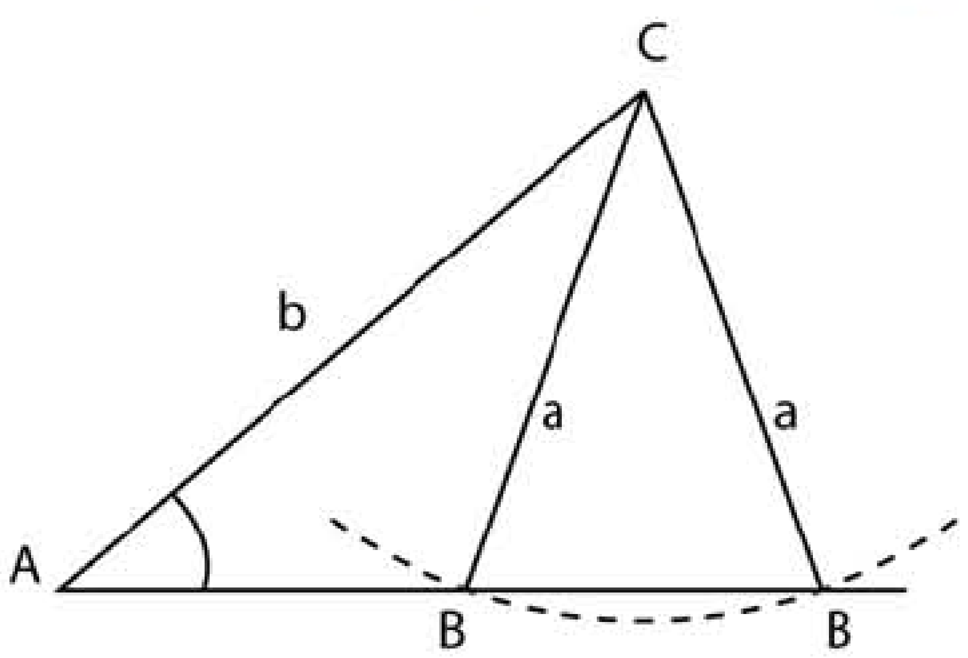
Outcome:
No solution

Case 2: When $a = b\sin A$
CB just touch the side opposite to C



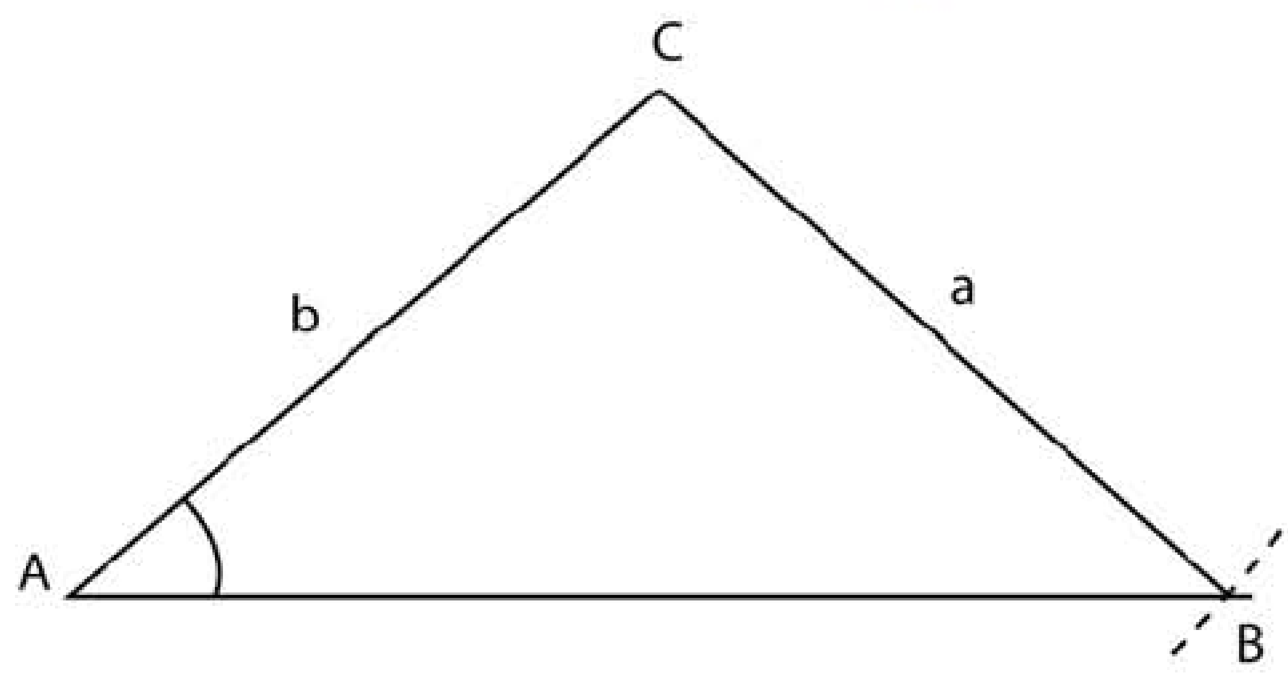
Outcome:
1 solution

Case 3: When $a > b\sin A$ but $a < b$.
CB cuts the side opposite to C at 2 points



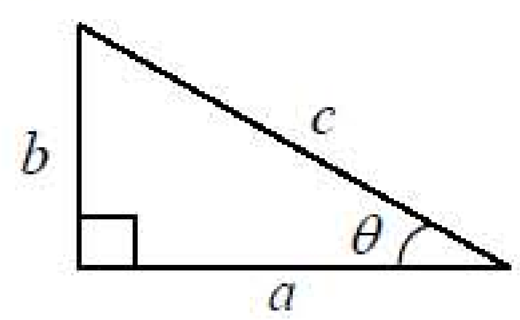
Outcome:
2 solution

Case 4: When $a > b\sin A$ and $a > b$.
CB cuts the side opposite to C at 1 points



Outcome:
1 solution

Useful information:



In a right angled triangle, you may use the following to solve the problems.

(i) Phythagoras Theorem: $c = \sqrt{a^2 + b^2}$

(ii) Trigonometry ratio:
$\sin\theta = \frac{b}{c}$, $\cos\theta = \frac{a}{c}$, $\tan\theta = \frac{b}{a}$

(iii) *Area* = ½ *(base)(height)*

11 Index Number

Price Index	**Composite index**
$$I = \frac{P_1}{P_0} \times 100$$ I = Price index / Index number P_0 = Price at the base time P_1 = Price at a specific time	$$\bar{I} = \frac{\Sigma W_i I_i}{\Sigma W_i}$$ $\bar{I}$ = Composite Index W = Weightage I = Price index
$$I_{A,B} \times I_{B,C} = I_{A,C} \times 100$$	